Math 8 Course Notes

Taught by Matt Porter

Youwen Wu

Winter 2024

> In the broad light of day mathematicians check their equations and their proofs, leaving no stone unturned in their search for rigour. But, at night, under the full moon, they dream, they float among the stars and wonder at the miracle of the heavens. They are inspired. Without dreams there is no art, no mathematics, no life.
>
> – Michael Atiyah

Contents

§1. Course Logistics

The textbook for the course is *Smith, Eggen, Andre. A Transition to Advanced Mathematics. 8th ed.* ISBN: `978-1-285-46326-1`. Chapters 1-5 will be covered.

Lecture meets every M-W-F from 12:00 – 12:50 in Phelps 1444. Recitation meets M-W from 7:00 – 7:50 in HSSB 1236.

§1.1. Homework

Homework is from textbook and is worth 30% of the grade, due on Gradescope. Homework is due every W at 11:59 PM. LaTeX is recommended for typesetting but of course we will be using Typst, the superior typesetting software for mathematics.

Section and problem numbers should be clearly labeled and problems should be done on a single column.

The lowest homework score will be dropped.

§1.2. Exams

Each exam is 20% of the grade. The final exam wil replace the lowest of the first two exam scores if it is higher.

§2. Chapter 1: Logic and Proofs

§2.1. Trivial Preliminaries

Definitions barely worth considering. Included purely for posterity.

Definition 2.1.1 *(Proposition)*. A proposition is a sentence which is either true or false.

Example (Primes).
The numbers 5 and 7 are prime.

Example (Not a proposition).
$x^2 + 6x + 8 = 0$

Propositions may be stated in the formalism of mathematics using connectives, as **propositional forms**.

Definition 2.1.2 *(Propositional forms)*. Let P and Q be propositions. Then:

1. The conjunction of P and Q is written $P \wedge Q$ (P and Q).
2. The disjunction of P and Q is written $P \vee Q$ (P or Q) (here "or" is the inclusive or).
3. The negation of P is written $\neg P$.

Definition 2.1.3 *(Tautology)*. A propositional form for which all of its values are true. In other words, a statement which is always true.

Definition 2.1.4 *(Contradiction)*. A propositional form for which all of its values are false. In other words, a statement which is always false.

Exercise 2.1.5 (Prove that $(P \vee Q) \vee (\neg P \wedge \neg Q)$ is a tautology).
Trivial, omitted.

Example (Several denials of the statement "integer n is even").
- It is not the case that integer n is even.
- Integer n is not even.
- $n \neq 2m, \forall m \in \mathbb{Z}$
- $n = 2m + 1, \exists m \in \mathbb{Z}$

DeMorgan's Laws tell us how to distribute logical connectives across parentheses.

Theorem 2.1.6 (DeMorgan's Laws).
1. $\neg(P \vee Q) = \neg P \wedge \neg Q$
2. $\neg(P \wedge Q) = \neg P \vee \neg Q$

Proof. Trivially, by completing a truth table. □