Pendulum Lab

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1 Introduction

The motion of a simple pendulum and the acceleration due to local gravity are connected, and we can express its period T in terms of its length ℓ and gravity g through the formula we derived:

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

This also means that if we can determine T and ℓ, we can obtain a value for g. In this paper, we will show how to calculate an experimental value of g for Earth's gravity using a simple pendulum.

2 The Experiment

2.1 Materials

- Ring stand
- String
- Weight (with known mass)
- Meterstick
- Timer
- Camera

2.2 Procedure

1. Set up the ring stand and hang the weight from the ring with a piece of string. Record the length of the string.
2. Set up the stopwatch and camera (recording, preferably in slow motion) so that they can see the pendulum.
3. Start the stopwatch as you release the weight from its initial position.
4. Wait for 10 oscillations, and check the recording to determine the time it took for those 10 oscillations. Collect this data.
5. Repeat 2 more times, for a total of 3 trials, and record the average.
6. Change the length of the string and repeat the trials again. Run the experiment for 3 different lengths of string.

2.3 Diagram

TODO

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3 Data

3.1 Measurements

$\ell = 0.28\text{m}$

Trial	Time, 10 oscillations (s)	Period (s)
1	10.82	1.082
2	10.76	1.076
3	10.32	1.032
Average	10.63	1.063

$\ell = 0.18\text{m}$

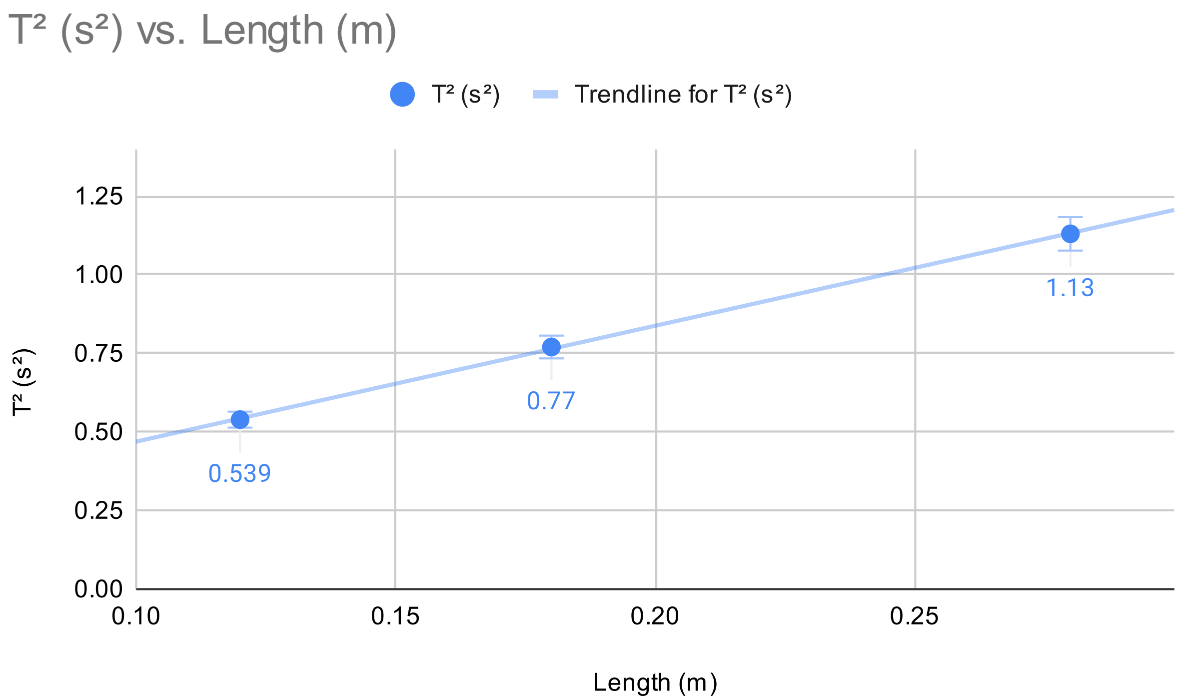
Trial	Time, 10 oscillations (s)	Period (s)
1	8.76	0.876
2	8.76	0.876
3	8.81	0.881
Average	8.78	0.878

$\ell = 0.12\text{m}$

Trial	Time, 10 oscillations (s)	Period (s)
1	7.33	0.733
2	7.38	0.738
3	7.32	0.732
Average	7.34	0.734

3.2 Chart

The average periods (T) for each length have been plotted below, as well as a best-fit trend line. The slope of this best-fit line is critical in our calculation of g, as shown in the next section.



$$\text{Slope } (m) = 3.684$$

4 Calculations

4.1 Calculating acceleration due to gravity

From our formula:

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$T^2 = 4\pi^2 \cdot \frac{\ell}{g}$$

$$\frac{T^2}{\ell} = \frac{4\pi^2}{g} \Rightarrow \boxed{g = \frac{4\pi^2}{m}}$$

where m is the slope of the graph of T^2 versus ℓ, as pictured above. Plugging in our value of $m = 3.684$, we get

$$\boxed{g = 10.72\frac{\text{m}}{\text{s}^2}}$$

4.2 Calculating error

The true value of gravity on Earth is around $9.81\frac{\text{m}}{\text{s}^2}$, so our experimental value is off by $0.91\frac{\text{m}}{\text{s}^2}$, a percent error of 9.3%.

4.3 Approximating error in our time measurements

Ideally, the period of the pendulum should be

$$2\pi \cdot \sqrt{\frac{0.28}{1.06}} = 1.06\text{s}$$

To estimate the amount of error, E_t, we made in our time measurements, we can use differentials. The error in our measurement would propagate through our calculations, resulting in our propagated error of $0.91\frac{\text{m}}{\text{s}^2}$. We can find the original E_t by differentiating the equation and plugging in our error to $\mathrm{d}g$. Note that we assume our measurements of ℓ to have negligible error.

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$g = \frac{4\ell\pi^2}{T^2}$$

$$\frac{\mathrm{d}g}{\mathrm{d}T} = -\frac{8\ell\pi^2}{T^3}$$

$$\mathrm{d}g = -\frac{-8\ell\pi^2}{T^2}\,\mathrm{d}T$$

$$T = \frac{t}{10} \;\Rightarrow\; \mathrm{d}t = 10\,\mathrm{d}T$$

$$\mathrm{d}t = -\frac{T^2}{\ell} \cdot \frac{\mathrm{d}g}{8\pi^2} \cdot 10$$

Note that $\frac{T^2}{\ell}$ is the slope $m = 3.684$, so

$$\mathrm{d}t = -\frac{36.84}{8\pi^2} \cdot \mathrm{d}g$$

Plugging in $\mathrm{d}g = 0.91$, we obtain

$$\mathrm{d}t = -0.42$$

meaning that

$$\boxed{E_t \approx |\,\mathrm{d}t| = 0.42\text{s}}$$

5 Conclusion

5.1 Sources of error

As shown above, we have an error of around 0.42s in our time measurements. Possible explanations for this error are

- Inaccurate timing due to human error
- Non-negligible torsion or friction in the string
- Non-negligible air resistance
- Slight inaccuracies when measuring the length of the string

5.2 Possible improvements

In order to ameliorate our experiment to mitigate these sources of error, we could

- Use a more precise timing system (possibly involving robotic arms)
- Use a lighter string
- Ensure that the string is not twisted before the experiment starts, creating unnecessary torsion
- Hang the string from the ring stand in a way that avoids the friction between the string and the stand itself when swinging